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Kinematics for $\pi^- + p \rightarrow \pi^0 + n$

Input data:

$$m_{\pi^-} = 139.5675\text{MeV}, \qquad m_{\pi^0} = 139.5675\text{MeV}$$

$$m_{\rm p} = 938.27231\text{MeV}, \qquad m_{\rm n} = 939.56563\text{MeV}$$

$$\Delta\pi = 4.5937\text{MeV}, \qquad \Delta M_N = 1.293318\text{MeV}$$

Energy and momentum of the π^0:

$$E_{\pi^0} = 0.5(m_p + m_{\pi^-}) + \frac{(m_{\pi^0} - m_n)(m_{\pi^0} + m_n)}{2(m_p + m_{\pi^-})} = 137.856\text{MeV}$$

$$T_{\pi^0} = 2.882\text{MeV}, \quad p_{\pi^0} = 28.046\text{MeV/c}$$

Energy, momentum, and velocity of the neutron:

$$T_n = \Delta\pi - T_{\pi^0} - \Delta M_N = 0.418\text{MeV}, \quad p_n = 28.046\text{MeV/c}, \quad \beta_n = 0.0298$$

Times of flight for path lengths of 1 m:

$$TOF_n = 111.72ns, \qquad TOF_\gamma = 3.33ns$$

Let one of the π^0 decay gammas be detected in a tagger counter. Let the energy of this photon be k_1, and let its angle with respect to the beam line be θ_1. Let the other photon be detected at θ_2 with energy k_2. Let the neutron be detected at an angle of θ_n with respect to the beam. Angles on the tagger side of the beam line will be shown as positive and angles on the opposite side will be shown as negative. θ_{1n} denotes the opening angle between the tagger photon and the neutron and θ_{12} denotes the angle between the two photons. Since the energy of the second photon is defined with greatest precision by the opening angle θ_{1n} this angle is taken to be the independent variable for the kinematics given in Table I. In this table the angles for the second gamma and for the neutron, shown in the last two columns, are calculated on the assumption that the tagger counter is located at $\theta_1 = 40^0$. Using this table, the energy, k_2, is most sensitive to the angle θ_{1n} at the symmetry point, so that the conjugate neutron counter should be at 140^0, and the second gamma should be detected at $\theta_2 = -116.5^0$. I suggest that (40^0, -116.5^0, 140^0) would be a good choice for the central angles of the three detector arrays.

At the symmetry point the second photon energy varies at the rate of 0.25 MeV/degree with respect to the opening angle θ_{1n}. If this angle is measured with an accuracy of 2.5^0 the energy of the gamma ray is determined to 0.6 MeV. With the tagger counter and the neutron counter each located 1 m from the target, this requirement limits each of them to a full width of approximately 10 cm (to yield $\sigma = 0.6$ MeV).

It is also worth noting that for a rather large range of neutron angles, the opening angle between the two gammas is very close to the minimum value, 156.5^0. Thus each gamma tagger will be associated with a small part of the CsI array.

It will be difficult to set up a coincidence trigger that includes the neutron. There are two reasons, 1) the neutron time-of-flight is very long, and 2) the neutron signal will be

small and the background will be large. On the other hand, since the signal from a 70 MeV gamma in the tagger counter will be easy to find, it makes sense to include only the pi-stop and the tagger in the event trigger. This trigger should be rather clean if one puts a high discrimination level on the tagger. The trigger can be used to gate the ADC's and the TDC's for the CsI array and for the neutron counters. Off-line, one can select the events that include a signal at the appropriate time from one of the neutron detectors. The electronic set-up for this arrangement is relatively simple.

TABLE I

θ_{1n} deg.	k_1 MeV	k_2 MeV	$dk_1/d\theta_{1n}$ MeV/deg.	θ_{12} degrees	θ_n degrees	θ_2 degrees
0	54.9	83.0	0	180	40	-140
10	55.0	82.8	0.028	176.7	50	-136.7
20	55.5	82.4	0.057	173.3	60	-133.3
30	56.2	81.7	0.085	170.1	70	-130.1
40	57.2	80.7	0.113	167.1	80	-127.1
50	48.4	79.4	0.141	164.3	90	-124.3
60	60.0	77.9	0.167	161.8	100	-121.8
70	61.8	76.1	0.193	159.7	110	-119.7
80	63.8	74.0	0.216	158.1	120	-118.1
90	66.1	71.8	0.235	157.0	130	-117.0
100	68.5	69.4	0.248	156.5	140	-116.5
110	71.0	66.8	0.255	156.8	150	-116.8
120	73.6	64.3	0.252	157.8	160	-117.8
130	76.0	61.8	0.238	159.7	170	-119.7
140	78.3	59.6	0.212	162.4	180	-122.4
150	80.2	57.6	0.173	165.9	-170	-125.9
160	81.7	56.2	0.123	170.2	-160	-130.2
170	82.6	55.2	0.064	175.0	-150	-135.0
180	83.0	54.9	0	180	-140	-140