
PIBETA Note: **Extracting V_{ud} from $R_{\pi\beta}$**
 By: Dinko Počanić
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Since there have been inquiries regarding the details of the calculation of V_{ud} in our paper on the $\pi\beta$ branching ratio, I am including them below.

Including loop corrections, δ , the branching ratio of pion beta decay is given by [1, 2]:

$$R_{\pi\beta} = \frac{G_\mu^2 |V_{ud}|^2 \tau_{\pi^+}}{30\pi^3} \left(1 - \frac{\Delta}{2M_+}\right)^3 \Delta^5 f(\epsilon, \Delta)(1 + \delta), \quad (1)$$

where G_μ is the Fermi weak coupling constant, τ_{π^+} the pion mean lifetime, $\Delta = M_+ - M_0$, $\epsilon = (m_e/\Delta)^2$, M_+ , M_0 , and m_e are the masses of the π^+ , π^0 , and the electron, respectively, while f , the Fermi function, is given by

$$f(\epsilon, \Delta) = \sqrt{1 - \epsilon} \left[1 - \frac{9}{2}\epsilon - 4\epsilon^2 + \frac{15}{2}\epsilon^2 \ln \left(\frac{1 + \sqrt{1 - \epsilon}}{\sqrt{\epsilon}} \right) - \frac{3}{7} \frac{\Delta^2}{(M_+ + M_0)^2} \right]. \quad (2)$$

Now the values of the quantities used in the calculation, all from the Particle Data Group:

$$\begin{aligned} \frac{G_\mu}{(\hbar c)^3} &= 1.16639(1) \times 10^{-5} \text{ GeV}^{-2} \\ \hbar c &= 197.326960(8) \text{ MeV} \cdot \text{fm} \\ \tau_{\pi^+} &= 26.033(5) \text{ ns} \\ M_+ &= 139.5702(4) \text{ MeV}/c^2 \\ M_0 &= 134.9766(6) \text{ MeV}/c^2 \\ \Delta &= 4.5936(5) \text{ MeV}/c^2 \\ m_e &= 0.5109980(2) \text{ MeV}/c^2 \end{aligned}$$

Finally, for the radiative corrections we take the most precise model-independent evaluation by Cirigliano et al. [3]

$$\delta = 0.0334(10). \quad (3)$$

Using the above values, one gets, e.g.,

$$\epsilon = 0.0123746(27) \quad \text{and} \quad f(\epsilon, \Delta) = 0.941018(3). \quad (4)$$

Combining the above expressions and propagating the errors, we get the key relation

$$R_{\pi\beta} = 1.0947(1) \times 10^{-8} |V_{ud}|^2. \quad (5)$$

Substituting our result:

$$R_{\pi\beta} = 1.036 \pm 0.004(\text{stat}) \pm 0.005(\text{syst}), \quad (6)$$

we get the result quoted in our paper:

$$V_{ud} = 0.9728 \pm 0.0030. \quad (7)$$

References

1. G. Källèn, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass., 1964).
2. A. Sirlin, *Rev. Mod. Phys.* **50**, 573 (1978), [erratum: *ibid.*, **50**, 905 (1978)].
3. V. Cirigliano, M. Knecht, H. Neufeld, and H. Pichl, *Eur. Phys. J. C* **27**, 255 (2003).