A High Precision Measurement of the Pion Form Factors via Radiative Pion Decay $\pi^+ \to e^+ \nu \gamma$

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Abstract

In this paper we present the results of the PIBETA collaboration analysis of the radiative pion decay $\pi^+ \to e^+ \nu \gamma$. This decay was studied in a broad region of the kinematic phase space allowing for a precise measurement of the pion vector and axial vector form factors F_V and F_A as well as the dependence of the form factors on the invariant mass of the eν pair. If known accurately, these parameters impose stringent constraints on the CVC hypothesis and deviations from the $V-A$ form of the weak current.

1 Introduction to Pion Decays

As the lightest mesons, pions present a unique opportunity to study weak interactions of quarks. The ratio of the decay rates of the charged pion $\pi^+ \to \mu^+ \nu$ ($\pi_{\mu 2}$) to $\pi^+ \to e^+ \nu$ (π_{e2}) is one of the strongest manifestations of the $V-A$ form of the weak interaction. The pion beta, $\pi^+ \to \pi^0 e^+ \nu$ (π_β), decay rate provides a precise measurement of the V_{ud} element of the CKM matrix [1]. Finally, the radiative pion decay, $\pi^+ \to e^+ \nu \gamma$ ($\pi_{e2\gamma}$), offers a glimpse into the hadronic structure of the charged pion via detection of the real photon associated with the virtual hadronic states of the pion constituents [2].

2 The $\pi_{e2\gamma}$ Decay in the Standard Model

The Standard Model description of the $\pi_{e2\gamma}$ decay parametrizes the branching ratio of the decay with two *a priori* unknown form factors, i.e., the vector

form factor F_V and axial-vector form factor F_A, such that

$$\frac{dB_{\pi e2\gamma}^{\text{the}}}{dx\,dy} = \frac{\alpha}{2\pi}B_{\pi e2}\Big\{IB + \left(\frac{m_\pi^2}{2f_\pi m_e}\right)^2 \left[(F_V + F_A)^2\,SD^+ + (F_V - F_A)^2\,SD^-\right]$$
$$+ \left(\frac{m_\pi}{f_\pi}\right)\left[(F_V + F_A)\,S_{\text{int}}^+ + (F_V - F_A)\,S_{\text{int}}^-\right]\Big\} + \text{r.c.}\,, \qquad (1)$$

where α is the fine structure constant, m_π and m_e are pion and positron masses, and $f_\pi = 130.7\,\text{MeV}$ is the pion decay constant. The $IB\,(x,y)$, $SD^\pm\,(x,y)$, and $S_{\text{int}}^\pm\,(x,y)$ (IB–SD interference) terms depend on the kinematic variables $x = 2E_\gamma/m_\pi$ and $y = 2E_e/m_\pi$ where E_e and E_γ are the particle's energies, and "r.c." stands for radiative corrections. The conserved vector current (CVC) hypothesis [3] relates F_V to the π^0 lifetime, yielding $F_V = 0.0259(9)$ [5]. The value of the F_A is model dependent and chiral symmetry calculations (see Ref. [4] and references therein) give a value for F_A in the range 0.010–0.014. In the lowest order of the chiral expansion the ratio of the form factors is related to the pion electric polarizability $\alpha_E = (\alpha/8\pi^2 m_\pi F_\pi^2) \times F_A/F_V$.

3   The $\pi_{e2\gamma}$ Decay in Experiment

Most of the earlier experimental measurements of the $\pi_{e2\gamma}$ decay were conducted in the high-E_γ, high-E_e regions of the phase space and relied on the precise knowledge of the vector form factor from the neutral pion life time, thus looking for the ratio of the F_A/F_V. The world average prior to the PIBETA publications was $F_A = (116 \pm 16) \times 10^{-4}$ [5].

Our measurements were performed at the $\pi E1$ beam line at the Paul Scherrer Institute (PSI), Villigen, Switzerland, using a stopped π^+ beam and the PIBETA detector [6]. For this analysis we used 2.0×10^{13} stopped π^+'s and the decay products accumulated in three overlapping regions of the phase space defined as: A: $E_{e^+}, E_\gamma > 50\,\text{MeV}$; B: $E_{e^+} > 10\,\text{MeV}$ and $E_\gamma > 50\,\text{MeV}$; C: $E_{e^+} > 50\,\text{MeV}$ and $E_\gamma > 10\,\text{MeV}$, with relative angle $\theta_{e^+\gamma} > 40°$ for all regions.

In our experiment we have observed positron and photon pairs from the $\pi_{e2\gamma}$ decay in overdetermined kinematics which allowed for a strong suppression of the copious positron background coming from the decays of the muons. The time difference distribution between the positron and the photon arrival to the CsI calorimeter defined our peak signal in the $\Delta t = |t_e - t_\gamma| < 5\,\text{ns}$ region. The sample of the accidental background taken from the sidebands of the same distribution was subtracted from the events in the peak.

In parallel to the $\pi_{e2\gamma}$ events, we have recorded a large sample of the non-radiative π_{e2} decays. In order to reduce the systematic uncertainty associated with the absolute number of pions stopped in the target, we have used these decays for the absolute normalization of the radiative pion decay branching ratio. The π_{e2} absolute branching ratio is a very well known quantity both theoretically and experimentally [5,7,8]. Both types of events were corrected by the detector acceptance. The acceptance calculations were provided by a GEANT3 based rendition of the PIBETA detector. The quality of the data and the simulation is demonstrated in Fig. 1.
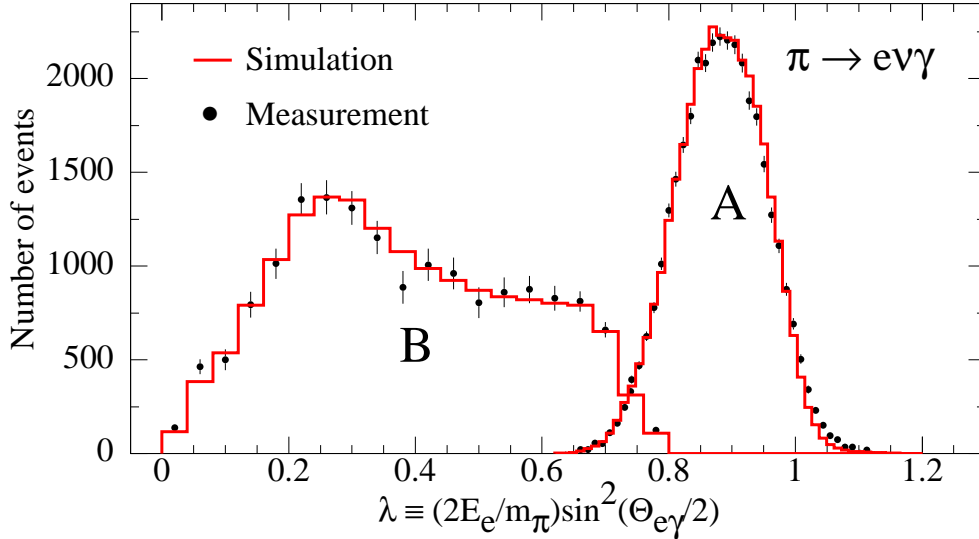

Figure 1: Background-subtracted $\pi^+ \rightarrow e^+\nu\gamma$ distribution of the kinematic variable $\lambda \equiv (2E_e/m_\pi)\sin^2(\theta_{e\gamma}/2)$ for regions B and A.

In order to extract the value of the form factors, experimental and theoretical branching ratios of the $\pi_{e2\gamma}$ decay were combined into the χ^2 function

$$\chi^2 = \sum_{i=A,B,C} \frac{(B_{\pi e2\gamma}^{\exp(i)}(F_A, FV, a) - B_{\pi e2\gamma}^{\text{the}(i)}(F_A, F_V, a))^2}{\sigma_i^2(F_A, F_V, a)}, \qquad (2)$$

where σ_i^2 are the combined uncertainties for each region, and the parameter a signifies the dependence of the form factor on the invariant mass of the $e\nu$ pair q^2 such that $F_V(q^2) = F_V(0)(1 + a \cdot q^2)$ and $F_A(q^2) = F_A(0)$. The χ^2 function was minimized as function of the free parameters F_A, F_V, and a.

4   Results

In this analysis we have reconstructed $35{,}948 \pm 194$ (0.54%) events in region A, $16{,}246 \pm 331$ (2.0%) events in region B and $13{,}263 \pm 161$ (1.2%) events in region C, where numbers in parentheses are fractional statistical uncertainties.

The simplest analysis of our data is in the form of a fit with fixed values of $F_V = 259 \times 10^{-4}$ and $a = 0.041$ [9],leaving only one free parameter, F_A. Under these conditions our data provide $F_A = (119 \pm 1) \times 10^{-4}$. This result represents a sixteenfold improvement in precision over the pre-PIBETA world average. Alternatively, we released all three parameters F_A, F_V, and a simultaneously, and obtained $F_A = (117 \pm 17) \times 10^{-4}$, $F_V = (258 \pm 17) \times 10^{-4}$, and $a = 0.095 \pm 0.058$. These results (i) agree very well with the predictions of the CVC hypothesis for the charged pion form factors, (ii) represent a fourteenfold improvement in the precision of the vector form factor F_V, and (iii) provide the first ever measurement of the charged pion form factor slope parameter a. Finally, fixing $F_V = 0.0259$, $a = 0.041$ (standard model values), and $F_A = 0.0119$ (our best-fit value), we varied the contribution of the putative tensor coupling form factor, F_T, as defined in Ref. [10]. The optimal fit yields $F_T = (-0.6 \pm 2.8) \times 10^{-4}$, or $-5.2 \times 10^{-4} < F_T < 4.0 \times 10^{-4}$ at the 90% confidence limit.

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