A survey of the rare pion and muon decays.

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Abstract

Our collaboration has used a detector system based on a non-magnetic pure CsI calorimeter at the Paul Scherrer Institute to collect the world's largest sample of rare pion and muon decays. We have measured the absolute $\pi^+ \to \pi^0 e^+ \nu$ decay branching ratio with a 0.55 % total uncertainty. The $\pi^+ \to e^+ \nu \gamma$ data set was used to extract weak axial and vector pion form factors F_A and F_V along with the branching ratios evaluated for broad kinematic regions. The $\mu^+ \to e^+ \nu \nu \gamma$ data were used to find the improved values of the Michel parameter $\bar{\eta}$ and to confirm the current value of the parameter ρ. These results bring major improvements in accuracy over the current Particle Data Group listings, and agree well with the predictions of the Standard Model.

1 Introduction

The PIBETA experiment at the Paul Scherrer Institute (PSI) is a collaboration of seven institutions that collected the world's largest sample of rare pion and muon decays during the several beam periods in 1999-2001 and 2004 [24].

The PIBETA detector is based on a large acceptance 240-module pure CsI electromagnetic shower calorimeter. The detector also includes (see Fig. 1) an active degrader AD, a segmented plastic target AT, a 20-bar cylindrical plastic scintillator hodoscope PV for particle identification, a pair of cylindrical multi-wire proportional chambers MWPC1/2 for charged particle tracking and an active cosmic veto detector (not shown) [14].

The incident 114 MeV/c π^+ beam with small e^+ and μ^+ contaminations was tagged with a thin forward beam counter (BC), slowed down in the

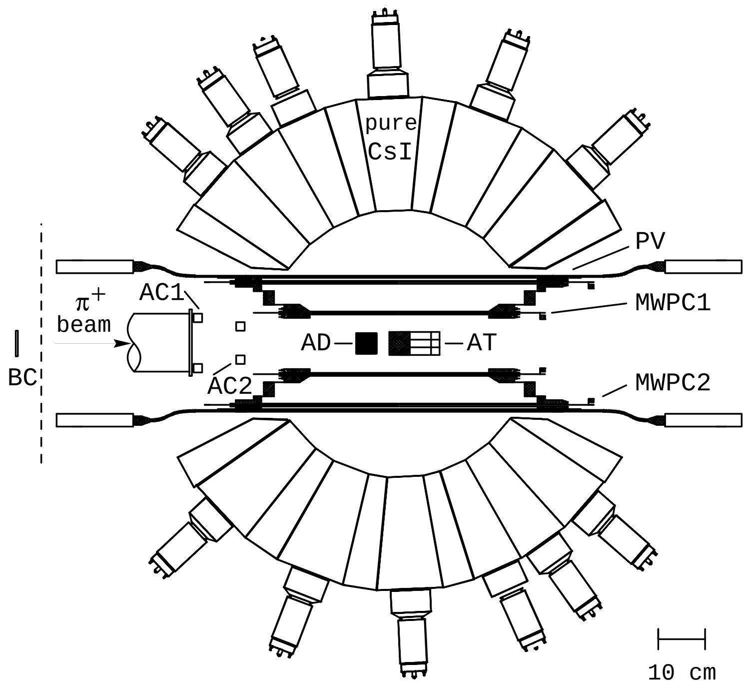


Figure 1: Pibeta detector: cross-sectional view showing the major detector sub-systems.

degrader (AD) and ultimately stopped in a tight $\simeq 9\,\mathrm{mm}^2$ spot in the active target (AT). The pions subsequently decayed at rest in the laboratory frame and the decay products were detected in the detector with geometric coverage close to 3π of the solid angle. The 1999-2001 runs, optimized for the pion beta decay measurement, used beam intensities of up to $10^6\,\pi^+/\mathrm{s}$. Lower beam intensities of 50-200 π^+/s were used in the year 2004, optimized for acquisition of radiative decay events.The recorded data, comprising 2.0×10^{13} π^+ stops, were obtained by a dedicated set of electronics triggers, some of which were prescaled.

This paper mainly discusses the following types of the pion and muon decays:

- $\pi^+ \to \mathrm{e}^+\nu$ and $\mu^+ \to \mathrm{e}^+\nu\bar{\nu}$ decays are discussed in section 2. These decays are abundant in our experiment and well understood theoretically. In order to reduce the systematic uncertainty related to the number of stopped pions, these decays were used to normalize the yields of the decays under study as follows:

$$B^{\mathrm{exp}}_{\mathrm{decay}} = B_{\mathrm{norm}} \cdot \frac{A_{\mathrm{norm}} \cdot N_{\mathrm{decay}}}{N_{\mathrm{norm}} \cdot A_{\mathrm{decay}}} \tag{1}$$

where B_{norm} is branching ratio of the normalizing decay, N_{decay} is the number of events detected for a given decay, A_{decay} is the acceptance for

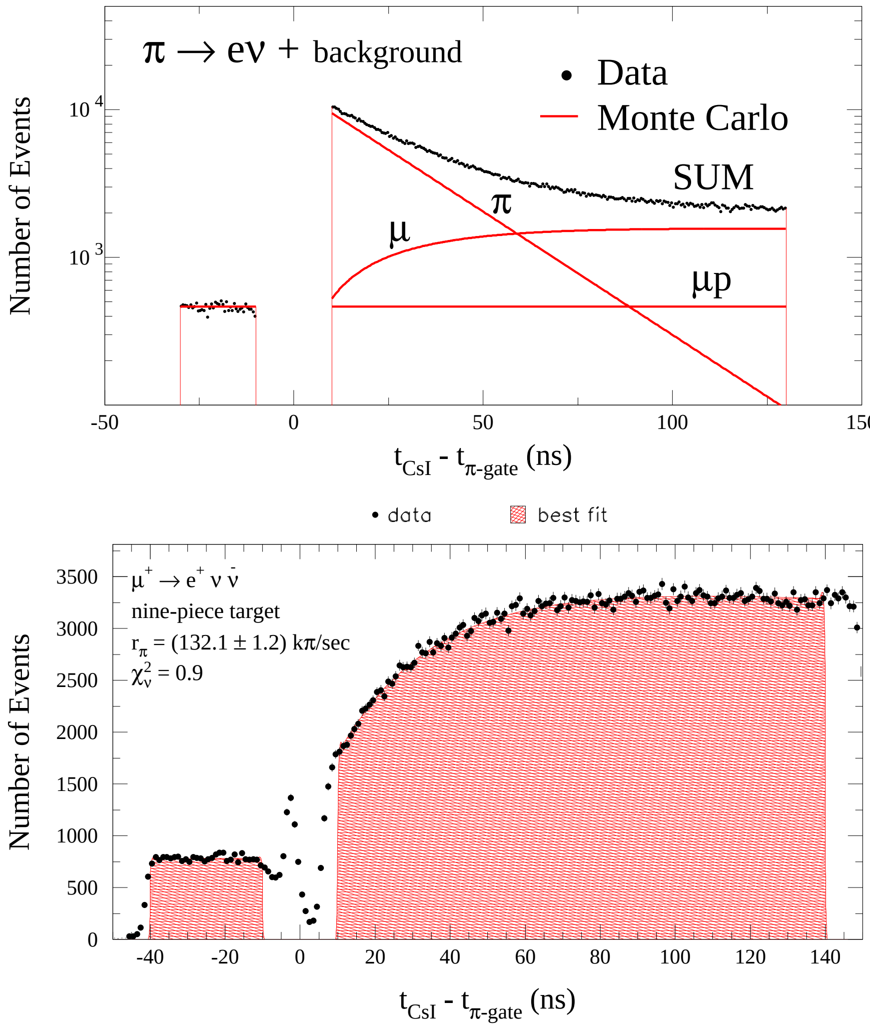


Figure 2: Time spectra of the positrons from $\pi^+ \to e^+\nu_e$ (top) and $\mu^+ \to e^+\nu\overline{\nu}$ (bottom) decays fit with the corresponding analytical functions of the decay probability. Total number of the detected events is obtained by integrating these time spectra.

the same decay. The $\pi^+ \to e^+\nu_e$ and $\mu^+ \to e^+\nu\overline{\nu}$ decays were used for normalization of the rare pion and muon decays, respectively.

- $\pi^+ \to \pi^0 e^+\nu_e$ is the original main focus of the experiment and is described in section 3.
- $\pi^+ \to e^+\nu_e\gamma$ decay description and relative discussions are in section 4.
- $\mu^+ \to e^+\nu\overline{\nu}\gamma$ process and results are given in section 5

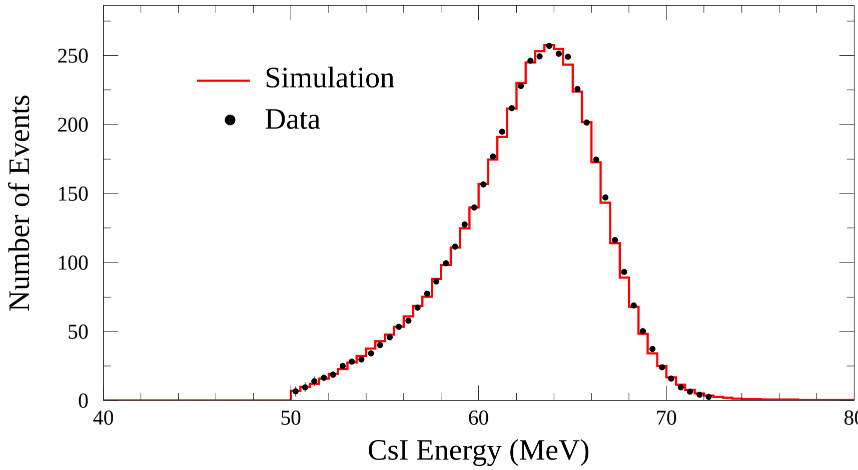


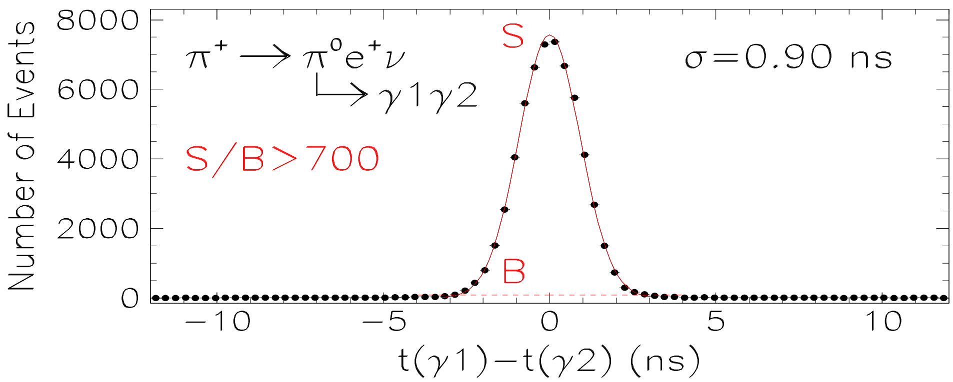
Figure 3: The match between the energy spectrum of the $\pi^+ \to \mathrm{e}^+\nu_\mathrm{e}$ positron in data (dots) and the Monte Carlo simulation (solid line).

2 Normalizing decays

Physics of the $\pi^+ \to \mathrm{e}^+\nu_\mathrm{e}$ [2] and $\mu^+ \to \mathrm{e}^+\nu\overline{\nu}$ [9], [20] is an extremely interesting subject in itself. In particular, several new experiments will soon study the $\pi^+ \to \mathrm{e}^+\nu_\mathrm{e}$ decay rate with a greater degree of accuracy [12]. For the purposes of this report we need to understand the total number of these events produced in our detector. In order to extract the number of detected decays we use the time spectra of the positrons registered in the CsI calorimeter. Fig. 2 shows the quality of the fits. To reconstruct the full response of our detector we used a GEANT3 based simulation of the PIBETA detector. In particular, Fig. 3 shows the match between the simulated and detected shapes of the positron energy spectrum for the $\pi^+ \to \mathrm{e}^+\nu_\mathrm{e}$ decay. These precise methods allowed us to calculate the normalizing constants $A_\mathrm{norm}/N_\mathrm{norm}$ in Eq. 1 for the subsequent use in the data analyses with a $< 0.5\%$ precision.

3 Pion beta decay

Pion beta decay $\pi^+ \to \pi^0 e^+\nu$ (π_β) rate offers one of the most precise means of testing the conserved vector current hypothesis (CVC) [10] and studying the weak u-d quark mixing [4]. The Standard Model (SM) description of the π_β decay is theoretically unambiguous within a 0.1 % uncertainty [16, 5], but a small $\sim 1 \times 10^{-8}$ branching ratio poses significant experimental challenges. The 3.9 % uncertainty of the previous most precise measurement, made using



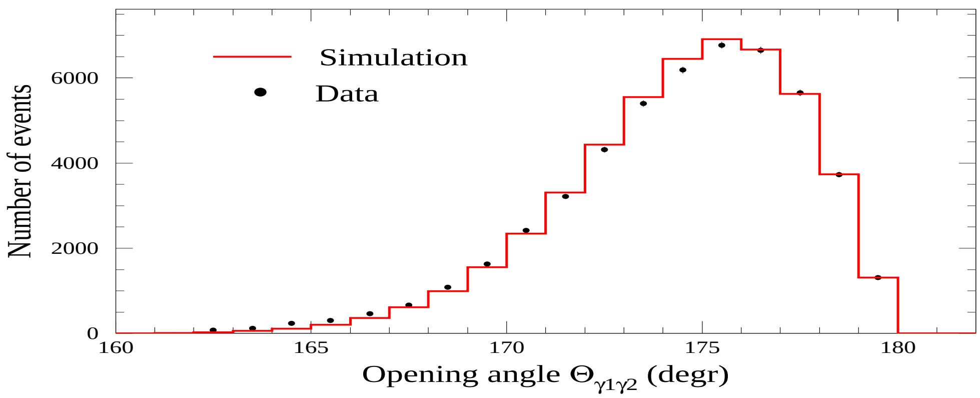


Figure 4: Representative π_β decay experimental spectra: signal-to-background ratio (top), and measured and simulated opening angle between the secondary photons produced by the neutral pion from the π_β decays (bottom).

components of the π^0 spectrometer at LAMPF [19], was not accurate enough to test the validity of radiative corrections which stand at $\sim 3\,\%$ [17].

Several fast analog triggers were designed to accept nearly all non-prompt π_β and and a sample of prescaled $\pi^+ \rightarrow e^+\nu$ events with individual shower energy exceeding the Michel endpoint (high threshold $\simeq$ 52 MeV).

The data analysis provided clean distributions of more than 64,000 π_β decay events which agreed very well with energy, angular and timing spectra predicted by the GEANT3 Monte Carlo detector simulations. The cosmic muon, prompt, radiative pion and accidental backgrounds were determined to be $< 1/700$ of the π_β signal, Fig. 4.

We have normalized the π_β yield to the yield of $\pi^+ \rightarrow e^+\nu$ events whose branching ratio is known with $0.33\,\%$ uncertainty experimentally [1, 6] and $\leq 0.05\,\%$ accuracy theoretically [18, 7]. Using the PDG [26] recommended value of $B^{\rm exp}_{\pi\rightarrow e\nu} = 1.230(4) \times 10^{-4}$, we find the pion beta branching ratio [23]:

$$B^{\rm exp}_{\pi\beta} = [1.036 \pm 0.004(\text{stat.}) \pm 0.005(\text{syst.})] \times 10^{-8}. \qquad (2)$$

When normalizing to the theoretical value $B^{\rm the}_{\pi\rightarrow e\nu} = 1.2353 \times 10^{-4}$ [18] we

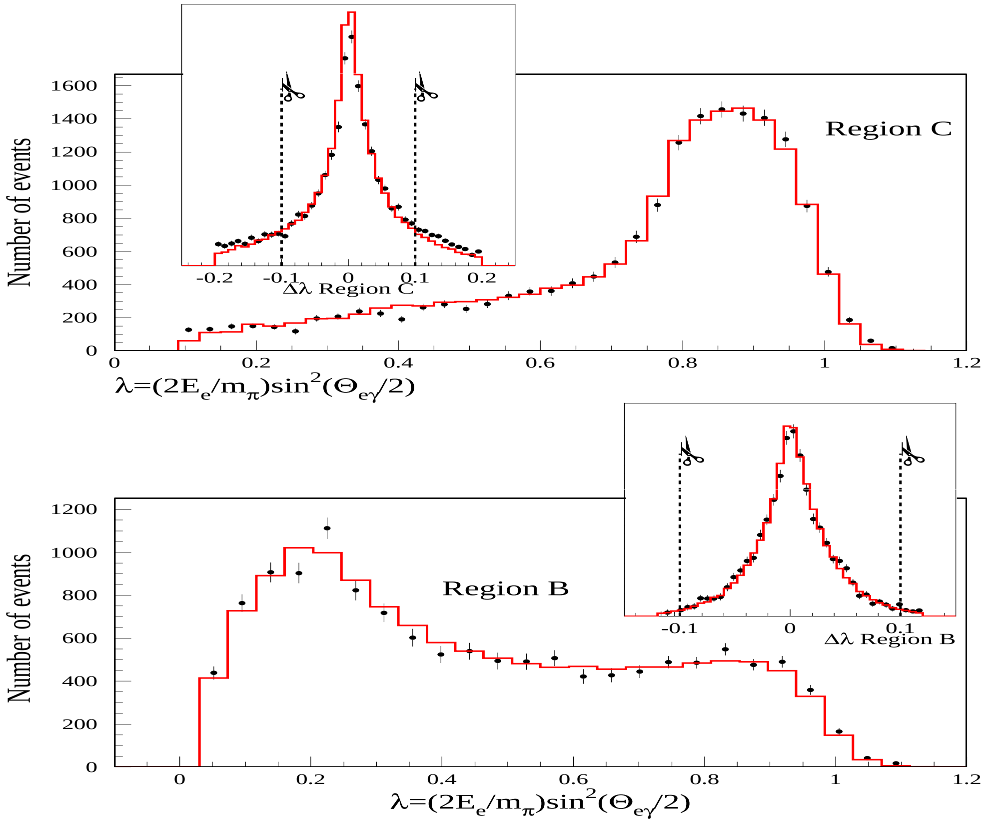


Figure 5: Select spectra for $\pi^+ \rightarrow \mathrm{e}^+\nu_\mathrm{e}\gamma$ decay: kinematic variable $\lambda = 2E_\mathrm{e}/m_\pi \sin^2(\theta_{\mathrm{e}\gamma/2})$ which is redundantly measured in our experiment as $\lambda^{'} = (E_\gamma + E_\mathrm{e} - m_\pi/2)/E_\gamma$ for regions C (top) and B (bottom). Inserts show plots of $\Delta\lambda = \lambda - \lambda^{'}$

obtain:

$$B_{\pi\beta}^{\mathrm{exp}} = [1.040 \pm 0.004(\mathrm{stat}) \pm 0.005(\mathrm{syst})] \times 10^{-8}. \quad (3)$$

Our result for $B_{\pi\beta}^{\mathrm{exp}}$ is in excellent agreement with the prediction of the SM:

$$B_{\pi\beta}^{\mathrm{SM}} = (1.038 - 1.041) \times 10^{-8}, \quad (4)$$

and stands as the most accurate confirmation of the CVC in a meson system to date. The values of the V_{ud} mixing element of the CKM extracted from the π_β rate measurements are

$$V_{ud} = 0.9728 \pm 30 \quad \text{or} \quad V_{ud} = 0.9748 \pm 25\ , \quad (5)$$

normalized to the experimental or theoretical value of the $B_{\pi\rightarrow e\nu}$ respectively.

4 Radiative pion decay

Precise measurement of the radiative pion decay $\pi^+ \rightarrow e^+\nu\gamma$ (RPD) branching ratio provides an excellent source of information on the value of F_A and

F_V, the weak axial and vector π^+ form factors respectively, together with limits on the non-$(V-A)$ contributions to Standard Model Lagrangian [13].

We have recorded radiative pion events in three overlapping phase space regions: (1) A: restricted to e^+-γ coincident pairs while energies of both particles $E_{e^+,\gamma} > 50.0\,\mathrm{MeV}$, and opening angle $\theta_{e^+\gamma} > 40.0°$, (2) B: events with positron energy $E_{e^+} > 10.0\,\mathrm{MeV}$, photon energy $E_\gamma > 50.0\,\mathrm{MeV}$, and opening angle $\theta_{e^+\gamma} > 40.0°$, and (3) C: events with photon energy $E_\gamma > 10.0\,\mathrm{MeV}$, positron energy $E_{e^+} > 50.0\,\mathrm{MeV}$, and opening angle $\theta_{e^+\gamma} > 40.0°$.

Of the three, region A is the most sensitive to the structure parameters of the pion. Complementary regions B and C allow us to resolve the quadratic ambiguity of the form factor solution [2], [3]. In addition, region C enabled us to determine for the first time the dependence of the pion form factors on the momentum squared transfered to the lepton pair. Based on the chiral perturbation theory ansatz [15], this dependence was linearly parametrized by an unknown factor a such that $F_V(q) = F_V(0)(1 + a \cdot q^2)$ and F_A remains a constant. Fig. 5 shows the match between the Monte Carlo simulation of our detector and the select kinematic distributions extracted from the data stream.

Theoretical predictions for the value of the form factors are somewhat model dependent. The vector form factor F_V, however, is well described by the CVC hypothesis as follows:

$$|F_V| \stackrel{\mathrm{cvc}}{=} \frac{1}{\alpha}\sqrt{\frac{2\hbar}{\pi\tau_{\pi^0}m_\pi}} = (259 \pm 9) \times 10^{-4} \ . \tag{6}$$

Therefore, in our main analysis we fixed the value of F_V and left the value of parameter F_A free. This method yielded the values

$$F_A(0) = (116 \pm 1) \times 10^{-4} \quad \text{and} \tag{7}$$

$$a = 0.066 \pm 0.020 \ . \tag{8}$$

The ratio of the χ^2 of the $F_A > 0$ to the $F_A < 0$ solution was 1:600, thus strongly favoring the positive solution. Using these best values of the form factors we have calculated the values of the absolute branching ratios summarized in Table 1.

The addition of a hypothetical tensor interaction term to the decay rate amplitude resulted in the upper limit of

$$|F_T(0)| \leq 5.1 \times 10^{-4} \tag{9}$$

at the 90 % confidence limit [3]. This limit is more than an order of magnitude smaller than the ISTRA collaboration re-analysis result [22].

Table 1: The optimal values of the absolute branching ratios of the radiative pion decay for different kinematic configurations. The value of $F_V = 0.0259$ is fixed while allowing free variations of F_A.

Region	$B^{\rm exp}_{\rm RPD}(10^{-8})$	$B^{\rm the}_{\rm RPD}(10^{-8})$	Number of events (10^3)
A	2.568 ± 0.019	2.579 ± 0.001	34.5
B	14.27 ± 0.22	14.44 ± 0.01	19.5
C	38.02 ± 1.01	37.48 ± 0.03	26.0

In addition, we have made an independent preliminary measurement of the pion vector form factor F_V which stands at:

$$F_V = (262 \pm 11) \times 10^{-4} \ . \tag{10}$$

These results represent improvements by an order of magnitude in the form factor determination and the F_V momentum dependence was measured for the first time for the pions.

5 Radiative muon decay

The radiative muon decay $\mu^+ \to e^+\nu\nu\gamma$ (RMD) measurement provides another critical consistency check of the overall analysis. In the Standard Model this process is parameterized via the Michel parameters all of which, save $\bar{\eta}$, can be determined from the ordinary muon decay [21]. A non-zero value of $\bar{\eta}$ would imply the non-$(V-A)$ structure of the electroweak interaction.

The most recent direct measurement of $\bar{\eta}$ can be interpreted as an upper limit of 0.141 (at 90 % CL) [8].

Our two-dimensional Michel parameter fit $(\rho, \bar{\eta})$ of the 4.2×10^5 radiative muon events collected in 2004 corresponds to the upper limit

$$\bar{\eta} \leq 0.033 \quad 68\% \text{ CL} \ , \quad \bar{\eta} \leq 0.060 \quad 90\% \text{ CL} \ , \tag{11}$$

and simultaneously yields the SM value $\rho = 0.751 \pm 0.010$ [25]. The details are summarized in Table 2.

We have also extracted the absolute branching ratio for the $\mu^+ \to {\rm e}^+\nu\bar{\nu}\gamma$ in the kinematic region $(E_\gamma > 10\,\text{MeV},\ \theta > 30^\circ)$

$$B^{\rm exp}_{\rm RMD} = [4.40 \pm 0.02 \ (\text{stat.}) \pm 0.09 \ (\text{syst.})] \times 10^{-3} \tag{12}$$

$$B^{\rm the}_{\rm RMD} = 4.30 \times 10^{-3} \ , \tag{13}$$

which agrees with SM prediction and constitutes a 15× increase in the precision over previous measurements.

Table 2: The optimal values of parameters ρ and $\bar{\eta}$ in the radiative muon decay: two-dimensional fit (top) and the fit with ρ fixed at the Standard Model value (bottom).

$\bar{\eta}$	ρ
$-0.081 \pm 0.054(\text{stat.}) \pm 0.034(\text{syst.})$	0.751 ± 0.010
$-0.084 \pm 0.050(\text{stat.}) \pm 0.034(\text{syst.})$	0.75 (fixed at SM)

6 CONCLUSION

We have reported new and improved absolute branching ratios for the following rare decays: (1) $\pi^+ \to \pi^0 e^+ \nu$, (2) $\pi^+ \to e^+ \nu \gamma$, and (3) $\mu^+ \to e^+ \nu \nu \gamma$. The yields of $\pi^+ \to e^+ \nu$ and $\mu^+ \to e^+ \nu \nu$ decays that were used for normalization are also internally consistent when compared to the total measured number of decaying π^+'s and μ^+'s [11].

Our results confirm the CVC hypothesis in the π^+ system at the 0.55 % level, rule out the tensor contribution in the radiative π^+ decay with the form factor $|F_T| \leq 5.1 \times 10^{-4}$ (90 % CL), and set the new 90 % CL limit on the parameter $\bar{\eta} \leq 0.060$ in the radiative μ^+ decay.

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