An Independent test of the CVC hypothesis in the charged pion
via the radiative pion decay $\pi^+ \rightarrow e^+ \nu_e \gamma$

Maxim A. Bychkov, University of Virginia

- Brief review of rare pion decay modes
- Radiative pion decay: $\pi^+ \rightarrow e^+ \nu_e \gamma$ ($\pi e\gamma$) in more detail
- CVC connection to the neutral pion
- Results, predictions and conclusions

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Experiment R-04-01 (PIBETA) collaboration members:

V. A. Baranov, c W. Bertl, b M. Bychkov, a Yu.M. Bystritsky, c E. Frlež, a
N.V. Khomutov, c A.S. Korenchenko, c S.M. Korenchenko, c M. Korolija, f
T. Kozlowski, d N.P. Kravchuk, c N.A. Kuchinsky, c D. Mzhavia, c,e
D. Mekterović, a D. Počanić, a P. Robmann, g O.A. Rondon-Aramayo, a
A.M. Rozhdestvensky, c T. Sakhelashvili, b S. Scheu, g V.V. Sidorkin, c
U. Straumann, g I. Supek, f Z. Tsamalaidze, e A. van der Schaaf, g
B. A. VanDevender, a E.P. Velicheva, c V.V. Volnykh, c and Y. Wang a

a Dept of Physics, Univ of Virginia, Charlottesville, VA 22904-4714, USA
b Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland
c Joint Institute for Nuclear Research, RU-141980 Dubna, Russia
d Institute for Nuclear Studies, PL-05-400 Swierk, Poland
e IHEP, Tbilisi, State University, GUS-380086 Tbilisi, Georgia
f Rudjer Bošković Institute, HR-10000 Zagreb, Croatia
g Physik Institut der Universität Zürich, CH-8057 Zürich, Switzerland
**Known and Measured Pion and Muon Decays** (PDG 2006)

<table>
<thead>
<tr>
<th>Decay</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \rightarrow \mu^+ \nu$</td>
<td>$0.9998770 (4)$</td>
</tr>
<tr>
<td>$\mu^+ \nu \gamma$</td>
<td>$2.00 (25) \times 10^{-4}$</td>
</tr>
<tr>
<td>$e^+ \nu$</td>
<td>$1.230 (4) \times 10^{-4}$</td>
</tr>
<tr>
<td>$e^+ \nu \gamma$</td>
<td>$1.61 (23) \times 10^{-7}$</td>
</tr>
<tr>
<td>$\pi^0 e^+ \nu$</td>
<td>$1.025 (34) \times 10^{-8}$</td>
</tr>
<tr>
<td>$e^+ \nu e^+ e^-$</td>
<td>$3.2 (5) \times 10^{-9}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 \rightarrow \gamma \gamma$</td>
<td>$0.98798 (32)$</td>
</tr>
<tr>
<td>$e^+ e^- \gamma$</td>
<td>$1.198 (32) \times 10^{-2}$</td>
</tr>
<tr>
<td>$e^+ e^- e^+ e^-$</td>
<td>$3.14 (30) \times 10^{-5}$</td>
</tr>
<tr>
<td>$e^+ e^-$</td>
<td>$6.2 (5) \times 10^{-8}$</td>
</tr>
<tr>
<td>$\mu^+ \rightarrow e^+ \nu \bar{\nu}$</td>
<td>$\sim 1.0$</td>
</tr>
<tr>
<td>$e^+ \nu \bar{\nu} \gamma$</td>
<td>$0.014 (4)$</td>
</tr>
<tr>
<td>$e^+ \nu \bar{\nu} e^+ e^-$</td>
<td>$3.4 (4) \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Physics Goals in Radiative Pion Decay

- Structure of the pion by measuring pion form factors. Independent measurement of pion polarizibility $\alpha_E$ with real photon
- Deviations from $V - A$ form of $L_{\text{weak}}$ by measuring absolute branching ratio
- Check of the CVC hypothesis by precise determination of the vector form factor $F_V$ and its momentum transfer dependence
- Comprehensive input to the $\chi$PT expansion coefficients
The PIBETA Experiment:
- stopped $\pi^+$ beam
- segmented active tgt.
- 240-det. CsI(p) calo.
- central tracking
- digitized PMT signals
- stable temp./humidity
- cosmic $\mu$ antihouse
Data Analysis: Method

In order to reduce the systematic uncertainties we use $\pi^+ \rightarrow e^+ \nu_e \ (\pi e_2)$ decay for normalization:

$$B_{\text{exp}}^{\text{decay}} = B_{\pi e_2} \cdot \frac{A_{\pi e_2} \cdot N_{\text{decay}}}{N_{\pi e_2} \cdot A_{\text{decay}}}$$

$B(A)_{\pi e_2}$ is branching ratio (acceptance) of $\pi^+ \rightarrow e^+ \nu_e$ decay

$N_{\text{decay}}$ is the number of events detected for a given decay in question

$A_{\text{decay}}$ is the acceptance for the same decay in question
$$\pi \rightarrow e \nu \gamma$$:

*Standard IB and V – A terms*

A tensor interaction, too?

Exchange of S=0 leptoquarks

P Herczeg, PRD 49 (1994) 247
\[ \pi \rightarrow e\nu\gamma: \text{Differential Branching Ratio} \]

\[
\frac{d^2 B^{\text{theor}}}{dxdy} = \frac{d^2 B_{IB}}{dxdy} + \frac{d^2 B_{SD}}{dxdy} + \frac{d^2 B_{\text{int}}}{dxdy} = \frac{\alpha}{2\pi} \left[ B_{\pi \rightarrow e\nu} \right] \left\{ IB(x, y) + \right. \\
\left. \frac{m^2_\pi}{4m^2_e} \left( \frac{F_V}{f_\pi} \right)^2 [(1 + \gamma)^2 SD^+(x, y) + (1 - \gamma)^2 SD^-(x, y)] + \right. \\
\left. \frac{F_V}{f_\pi} [(1 + \gamma)SD^+_{\text{int}}(x, y) + (1 - \gamma)SD^-_{\text{int}}(x, y)] \right\},
\]

where \( IB, SD^\pm, SD^\pm_{\text{int}} \) are analytical functions of 
\[ x \equiv 2E_\gamma/m_{\pi^+} = 1 - q^2 \quad \text{and} \quad y \equiv 2E_e/m_{\pi^+} \quad \text{and} \quad \gamma = F_A/F_V \]

\[ F_A(q^2) = F_A(0) \text{ is the axial-vector form factor} \]
\[ F_V(q^2) = F_V(0)(1 + a \cdot (1 - x)) \text{ is the vector form factor} \]
Theoretical Description: $\pi^+ \rightarrow e^+ \nu_e \gamma$ Decay

\[ \frac{x}{m_\pi} = \frac{2E_\gamma}{m_\pi} \]

\[ y = \frac{2E_e}{m_\pi} \]

SD$^+ \times 10^4$

IB$^\approx 1.5$ at $(x,y) = (0.2, 0.8)$

SD$^- \times 10^4$

Leyenda
Available Data on Pion Form Factors

\[ |F_V| \xrightarrow{\text{cvc}} \frac{1}{\alpha} \sqrt{\frac{2\hbar}{\pi \tau_{\pi^0} m_\pi}} = 0.0259(9) . \]

<table>
<thead>
<tr>
<th>( F_A \times 10^4 )</th>
<th>reference</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>106 ± 60</td>
<td>Bolotov et al. (1990)</td>
<td>((F_T = -56 \pm 17))</td>
</tr>
<tr>
<td>135 ± 16</td>
<td>Bay et al. (1986)</td>
<td></td>
</tr>
<tr>
<td>60 ± 30</td>
<td>Piilonen et al. (1986)</td>
<td></td>
</tr>
<tr>
<td>110 ± 30</td>
<td>Stetz et al. (1979)</td>
<td></td>
</tr>
<tr>
<td>116 ± 16</td>
<td>world average (PDG 2004)</td>
<td></td>
</tr>
</tbody>
</table>

In \(SU(3)\) limit CVC also predicts slope \(a\) is the same as for \(\pi^0 \rightarrow \gamma\gamma^*\)

\[ a = 0.032 \pm 0.004 \text{ (PDG 2006)} \]
\[ \pi \to e\nu\gamma: \text{Pion form factors and polarizability in } \chi PT \]

To first order in \( \chi PT \) the pion weak form form factors fix:

\[
\frac{F_A}{F_V} = 32\pi^2 (l_9^r + l_{10}^r),
\]

while the pion polarizability is given by

\[
\alpha_E = \frac{4\alpha}{m_\pi F_\pi^2} (l_9^r + l_{10}^r),
\]

so that

\[
\alpha_E = \frac{\alpha}{8\pi^2 m_\pi F_\pi^2} \cdot \frac{F_A}{F_V} \approx 6.24 \times 10^{-4} \text{ fm}^3 \cdot \frac{F_A}{F_V}.
\]

[To resolve \( l_9 \) and \( l_{10} \) one needs

\[
\frac{1}{6} \langle r_{\pi}^2 \rangle = \frac{2}{F_\pi^2} l_9^r - \frac{1}{96\pi^2 F_\pi^2} \left( \ln \frac{m_\pi^2}{\mu^2} + \frac{1}{2} \ln \frac{m_K^2}{\mu^2} + \frac{3}{2} \right),
\]

w.a. 1.1\%; most accurate data, NA7 1986.; last revisited at SELEX in 2001]
Available Results: \( \pi^+ \rightarrow e^+ \nu_e \) Decay

Marciano and Sirlin, [PRL 71 (1993) 3629]:

\[
\frac{\Gamma(\pi \rightarrow e\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))}_{\text{calc}} = (1.2352 \pm 0.0005) \times 10^{-4}
\]

Decker and Finkemeier, [NP B 438 (1995) 17]:

\[
\frac{\Gamma(\pi \rightarrow e\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))}_{\text{calc}} = (1.2356 \pm 0.0001) \times 10^{-4}
\]

Experiment, world average (PDG 2006):

\[
\frac{\Gamma(\pi \rightarrow e\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))}_{\text{exp}} = (1.230 \pm 0.004) \times 10^{-4}
\]
Data Analysis 99-01: $\pi^+ \to e^+ \nu_e$
Data Analysis 04: $\pi^+ \rightarrow e^+ \nu_e$

- Simulation
- Data

$\pi \rightarrow e\nu + \text{background}$

- Data
- Monte Carlo

SUM $\mu$ $\pi$ $\mu\pi$

Number of Events

$\text{CsI Energy (MeV)}$

$t_{\text{CsI}} - t_{\pi\text{-gate (ns)}}$
Data Analysis: $\pi^+ \rightarrow e^+ \nu_e \gamma$

**Region A**
- $\Theta_{e\gamma}=180^0$
- $E_{e^+} > 50 \text{ MeV}$
- $E_{\gamma} > 50 \text{ MeV}$
- $\Theta_{e\gamma} > 40^\circ$
- $y=2E_{e^+}/m_{\pi^+}$
- $x=2E_{\gamma}/m_{\pi^+}$
- $z=2E_{\nu}/m_{\pi^+}$

**Region B**
- $\Theta_{e\gamma}=180^0$
- $E_{e^+} > 10 \text{ MeV}$
- $E_{\gamma} > 50 \text{ MeV}$
- $\Theta_{e\gamma} > 40^\circ$
- $y=2E_{e^+}/m_{\pi^+}$
- $x=2E_{\gamma}/m_{\pi^+}$
- $z=2E_{\nu}/m_{\pi^+}$

**Region C**
- $\Theta_{e\gamma}=180^0$
- $E_{e^+} > 50 \text{ MeV}$
- $E_{\gamma} > 10 \text{ MeV}$
- $\Theta_{e\gamma} > 40^\circ$
- $y=2E_{e^+}/m_{\pi^+}$
- $x=2E_{\gamma}/m_{\pi^+}$
- $z=2E_{\nu}/m_{\pi^+}$

**Other Diagrams**
- $2E_{e^+}/m_{\pi^+}$
- $2E_{\gamma}/m_{\pi^+}$
- $2E_{\nu}/m_{\pi^+}$
**Data Analysis Method:** \( \pi^+ \rightarrow e^+ \nu_e \gamma \)

To extract the values of the FF’s we minimize:

\[
\chi^2 = \sum_{i=A,B,C} \frac{(B^\text{exp}_i(F_A, F_V, a) - B^\text{the}_i(F_A, F_V, a))^2}{\sigma^2_i(F_A, F_V, a)}
\]

Both experimental and theoretical \( B \)s depend on \( F_A, F_V, a \).
Data Analysis 99-01: $\pi^+ \rightarrow e^+ \nu_e \gamma$

**Region A**
- Number of events: 4000
- $t(e)-t(\gamma)$ (ns)
- $P/B = 73$

**Region B**
- Number of events: 2000
- $t(e)-t(\gamma)$ (ns)
- $P/B = 1.6$

**Region C**
- Number of events: 1000
- $t(e)-t(\gamma)$ (ns)
- $P/B = 4$

Acc. bgr.
Data Analysis 04: \( \pi^+ \rightarrow e^+ \nu_e \gamma \)

Region A
- Number of events
- \( t(e)-t(\gamma) \) (ns)
- P/B = 145

Region B
- Number of events
- \( t(e)-t(\gamma) \) (ns)
- P/B = 9

Region C
- Number of events
- \( t(e)-t(\gamma) \) (ns)
- P/B = 29

Acc. bgr.
Data Analysis 99-01: $\pi^+ \rightarrow e^+ \nu_e \gamma$

\[ \lambda = \frac{2E_e}{m_\pi} \sin^2 \left( \frac{\Theta_e}{2} \right) \]

**Region A**

<table>
<thead>
<tr>
<th>Number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
</tr>
<tr>
<td>Data</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Opening angle $\Theta$ Region A (dgr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of events</td>
</tr>
<tr>
<td>135</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>
Data Analysis 99-01: \( \pi^+ \rightarrow e^+ \nu_e \gamma \)
Data Analysis 04: $\pi^+ \rightarrow e^+ \nu_e \gamma$

\[ \lambda = \frac{2E_e}{m_\mu} \sin^2 \left( \frac{\Theta_e}{2} \right) \]

Number of events

Region C

Region B

$\Delta \lambda$
Results of the combo (99-01+04) analysis

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a new exp. value of $F_V$</td>
<td>$0.0258 \pm 0.0018$</td>
</tr>
<tr>
<td>the value of $F_A$</td>
<td>$0.0121 \pm 0.0018$</td>
</tr>
<tr>
<td>first meas’t of $q^2$ dep.: $a$</td>
<td>$0.070 \pm 0.058$</td>
</tr>
<tr>
<td>improved limit on $</td>
<td>F_T</td>
</tr>
</tbody>
</table>

**ALTERNATIVELY**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>for fixed CVC value of $F_V$</td>
<td>$0.0259$</td>
</tr>
<tr>
<td>and fixed $q^2$ dep. $a$</td>
<td>$0.032$</td>
</tr>
<tr>
<td>improved value of $F_A$</td>
<td>$0.0121 \pm 0.0001$</td>
</tr>
</tbody>
</table>
Results of the combo (99-01+04) analysis

Using fixed $F_V = 0.0259$ and $a = 0.032$ we obtain

<table>
<thead>
<tr>
<th>$E_{e^+}^{\text{min}}$ (MeV)</th>
<th>$E_\gamma^{\text{min}}$ (MeV)</th>
<th>$\theta_{e\gamma}^{\text{min}}$ (°)</th>
<th>$B_{\text{exp}}$ ($\times 10^{-8}$)</th>
<th>$B_{\text{the}}$ ($\times 10^{-8}$)</th>
<th>no. of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50</td>
<td>−</td>
<td>2.612(20)</td>
<td>2.612</td>
<td>35.9 k</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>40</td>
<td>14.45(22)</td>
<td>14.46</td>
<td>16.2 k</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>40</td>
<td>37.85(46)</td>
<td>37.51</td>
<td>17.3 k</td>
</tr>
</tbody>
</table>

$\alpha_E = (2.91 \pm 0.10) \cdot 10^{-4} \text{ fm}^3$
Results of the combo (99-01+04) analysis

<table>
<thead>
<tr>
<th>Description</th>
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</tr>
</thead>
<tbody>
<tr>
<td>A new exp. value of $F_V$</td>
<td>$(258 \pm 17) \cdot 10^{-4}$</td>
</tr>
<tr>
<td>The value of $F_A$</td>
<td>$(117 \pm 17) \cdot 10^{-4}$</td>
</tr>
<tr>
<td>First meas't of $q^2$ dep.: $a$</td>
<td>$0.095 \pm 0.058$</td>
</tr>
<tr>
<td>Improved limit on $F_T$: $F_T \leq 3.0 \cdot 10^{-4}$ 90% CL</td>
<td></td>
</tr>
</tbody>
</table>

**Alternatively**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>For fixed CVC value of $F_V$</td>
<td>$259 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>And fixed $q^2$ dep. $a$</td>
<td>$0.041$</td>
</tr>
<tr>
<td>Improved value of $F_A$</td>
<td>$(119 \pm 1) \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Numerical value of $F_T$</td>
<td>$(-0.60 \pm 2.78) \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>
Results of the combo (99-01+04) analysis

Using fixed $F_V=0.0259$ and $a=0.041$ we obtain

<table>
<thead>
<tr>
<th>$E_{e+}^{\text{min}}$ (MeV)</th>
<th>$E_{\gamma}^{\text{min}}$ (MeV)</th>
<th>$\theta_{e\gamma}^{\text{min}}$ ($^\circ$)</th>
<th>$B_{\text{exp}}$ ($\times 10^{-8}$)</th>
<th>$B_{\text{the}}$ ($\times 10^{-8}$)</th>
<th>no. of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50</td>
<td>$-$</td>
<td>2.614(21)</td>
<td>2.599</td>
<td>35.9 $k$</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>$40^\circ$</td>
<td>14.46(22)</td>
<td>14.45</td>
<td>16.2 $k$</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>$40^\circ$</td>
<td>37.69(46)</td>
<td>37.49</td>
<td>13.3 $k$</td>
</tr>
<tr>
<td>$m_e/2$</td>
<td>10</td>
<td>$40^\circ$</td>
<td>73.86(54)</td>
<td>74.11</td>
<td>65.4 $k$</td>
</tr>
</tbody>
</table>

$\alpha_E = (2.87 \pm 0.10) \cdot 10^{-4} \text{ fm}^3$
Summary of Radiative Pion Decay Results

- We improved the precision of pion form factors $F_A$, $F_V$ and absolute $B^\text{exp}$ sixteen-, five- and tenfold respectively. Confirmed the sign of the ratio $F_A/F_V$.

- We have determined for the first time the momentum dependence of the charged pion FF’s.

- We set a new stringent upper limit on the tensor interaction; excellent agreement with SM.

- We provided a sub percent precision input data for the $\chi PT$.

- Our radiative $\pi$ results provide critical input in controlling the systematics of the approved $\pi \to e\nu$ (PEN) experiment, R-05-01.

- The PEN experiment will double the R-04-01 data set on radiative $\pi$ decays, with yet lower backgrounds.

- Possibility of observing the Dalitz version $\pi^+ \to e^+\nu e^+e^-$. 