Measurements of the wire chambers displacement

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January 4, 2002

1 Task

Find out possible displacement between the coordinate system of the CsI calorimeter and the coordinate system of the wire chambers.

2 Motivation

In current $\pi\beta$ experimental setup most of charged particles originated in the target traverse through two sets of cylindrical wire chambers and leave showers in spherically shaped calorimeter. It is assumed that the center of two collinear cylindrical chambers and the spherical calorimeter coincide. However, it might not be so and precise knowledge of possible displacement is necessery in order to fully understand the geometry of the detector.

3 Method

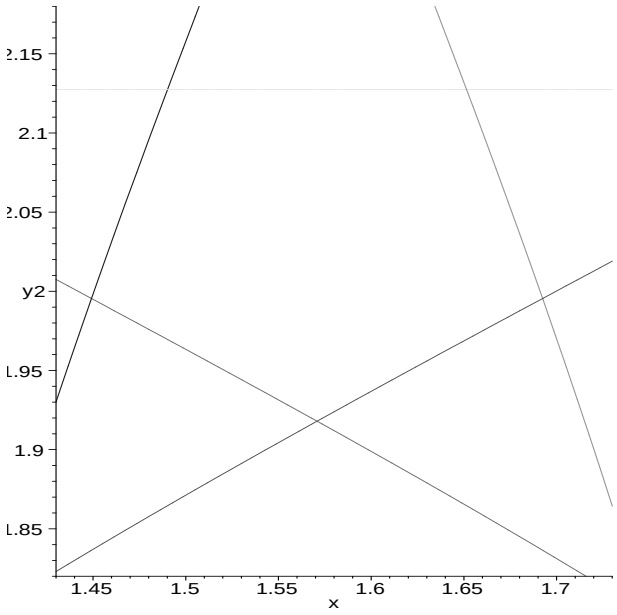
One could use raw chamber data to reconstruct a track left by a charged particle, thus viewing a track and its intercept with the calorimeter surface (a sphere of 26 cm radius) as "seen by the chamber". Additionally, finding the point where track hit the face of the calorimeter from the calorimeter data would enable us to compare these points and deduce possible displacement between them.

4 Implementation

In real life it is very complicated to find out the exact location of the point where track intercepts the face of the calorimeter from the calorimeter data. All we can say is whether one particular crystal was hit by a given track. Assuming that in such a case most of the energy were deposited in this crystal we can run the following procedure. Set our cuts to separate clean one charged particle events(Michels [1]), accept and reconstruct only one track and its interception with the calorimeter face. Then run through all the energies deposited in the calorimeter and find out the greatest value. Put calculated value of the intercept coordinate into histogram corresponding to the crystal with most energy deposited. An outcome of this procedure is a set of crystal's maps e.g. two dimensional plots (θ vs.φ) of the crystal illumination as seen by the chambers. Knowing the geometry of the crystal we can predict how θ and φ projection of the map should appear and compare them to experimental data.

Since radius of the calorimeter sphere is fixed at 26 cm we could effectively reduce our problem to two dimensional one. Namely everything depends only on spherical θ and φ of the crystals. We know that Michel events are uniformly distributed in space. Therefore if in θ-φ space our crystals were rectangular then probability density of hitting one particular crystal(for, say, theta projection) would be $\frac{dA}{d\theta}$ (where dA is infinitesimal area of the θ-φ projection) and therefore would be a constant. As could be seen from the picture our polygons remain polygons in θ-φ space and thus probability density is not a constant.

[1] $\mu \to e\nu_e\overline{\nu}_\mu$



Regular hexagon shape will remain hexagonal in θ-φ space. Obviously curves connecting the vertices are not straight lines but virtually indistinguishable from ones for our purposes

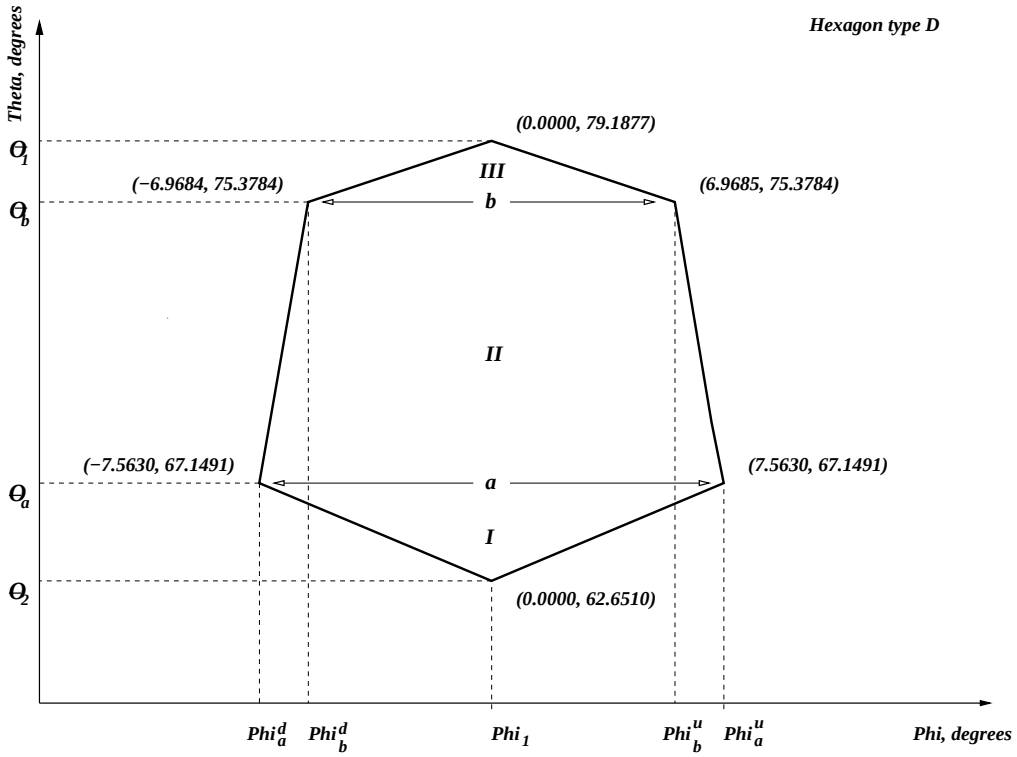
In addition to that our plots are smeared by the chamber resolution. Mathematically it means that our $\frac{dA}{d\theta}$ function is convoluted by some other function. Educated guess would be gaussian convolution and therefore trial function is

$$f(m) = \int_{-\infty}^{+\infty} \frac{dA(\theta)}{d\theta} \Gamma e^{-\frac{(\theta-m)^2}{\sigma^2}} d\theta$$

where Γ and σ are fit parameters of the gaussian.

As can be seen from the picture above unlike $\frac{dA}{d\theta}$, $\frac{dA}{d\varphi}$ function remains symmetric about the center of the crystal. So the center of the crystal related to the peak value of some symmetric bell-shape curve. It provides additional convenience not to calculate some complicated functions for our φ-projections and use some "build in" symmetric functions such as gaussian. Later results compared to the theoretical value confirm this assertion.

Detail calculations of $\frac{dA}{d\theta}$ are shown below



$$A^I(\theta) = 0.5(\theta - \theta_2)(\varphi^u - \varphi^d)$$

from line equation

$$\varphi^{u,d} = \theta \left(\frac{\varphi_a^{u,d} - \varphi_2}{\theta_a - \theta_2} \right) - \theta_2 \left(\frac{\varphi_a^{u,d} - \varphi_2}{\theta_a - \theta_2} \right) + \varphi_2$$

therefore

$$\varphi^u - \varphi^d = 2\theta\alpha - 2\theta_2\alpha = 2\alpha\,(\theta - \theta_2)$$

where

$$\alpha \equiv \frac{\varphi_a^u - \varphi_2}{\theta_a - \theta_2} = -\frac{\varphi_a^d - \varphi_2}{\theta_a - \theta_2} = 1.6814$$

is the tangent of corresponding straight line.[2].

[2]It yet to be proven that in θ-φ space these are the straight lines. It turns out to be very close

Thus

$$A^I = \alpha\left(\theta - \theta_2\right)^2, and\ \frac{dA^I}{d\theta} = 2\alpha\left(\theta - \theta_2\right)$$

By analogy

$$A^{II} = 0.5(\theta - \theta_a)(\varphi^u - \varphi^d + a), \quad \varphi^u - \varphi^d = -2\beta\theta + 2\beta\theta_a + a, \quad \beta \equiv \frac{\varphi_b^d - \varphi_a^d}{\theta_b - \theta_a} = 0.0723$$
$$A^{III} = 0.5(\theta - \theta_b)(\varphi^u - \varphi^d + b), \quad \varphi^u - \varphi^d = -2\gamma\theta + 2\gamma\theta_b + b, \quad \gamma \equiv \frac{\varphi_2 - \varphi_b^d}{\theta_1 - \theta_b} = 1.8293$$

And finally

$$\frac{dA^{II}}{d\theta} = -2\beta\theta + 2\beta\theta_a + a, \quad a = 15.126$$
$$\frac{dA^{III}}{d\theta} = -2\gamma\theta + 2\gamma\theta_b + b, \quad b = 13.937$$

Convolution with gaussian was performed in Maple. The outcome of this procedure is rather robust and won't be displayed here. It leaves us with 4 parameters of the fit to be determined. They are Γ and σ of the gaussian and θ_a, θ_b of the probability density. Also note that parameters θ_a, θ_b additionally enter the fitting function as limits of integrations over corresponding regions.

For my analysis I picked only crystal that were symmetric in φ. These turned out to be the following crystals: pentagons (crystals 0 through 9), hexagons type A (10 through 20), hexagons type B (80 through 90), hexagons type C (110 through 120) and hexagons type D(170 through 179). The following runs were analyzed

$run41024 - run41039$	$run41000 - run41022$	$run40690 - run40720$
$run42020 - run42032$	$run36030 - run36059$	$run36060 - run36079$
$run36870 - run36879$	$run36803, run40222$	$run40787$

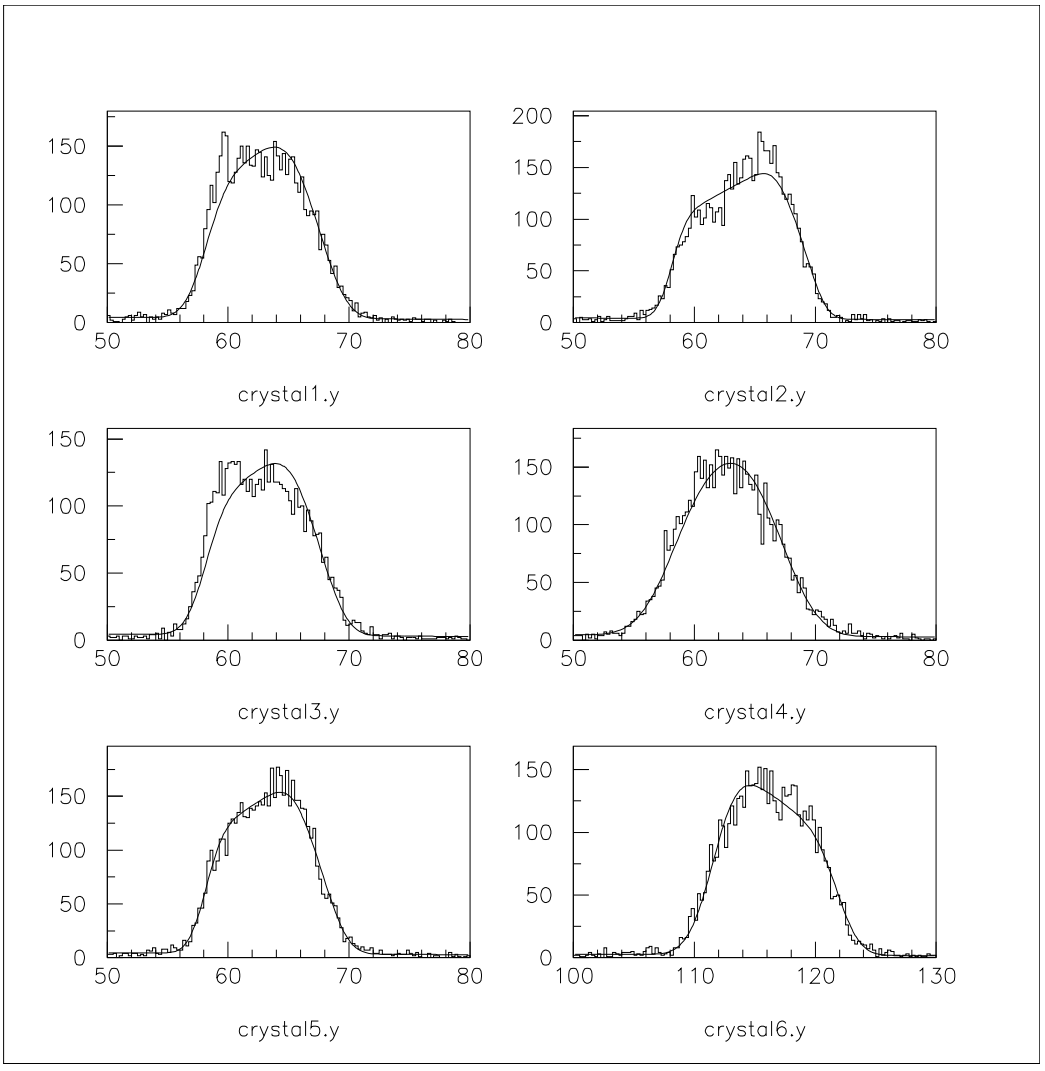
Unfortunately type C hexagons were located too close to the plastic vetos and their spectra were distorted by the vetoed showers leaking through the crystal towards the plastic veto.

Note "statistical advantage" of type D and B hexagons (chosen as an example for all shown calculations), since the trial function contains two physically interesting parameters (along with two parameters of the convoluting gaussian). Taking above calculations as a template it is easily seen than the rest of the crystals provide only one interesting point.

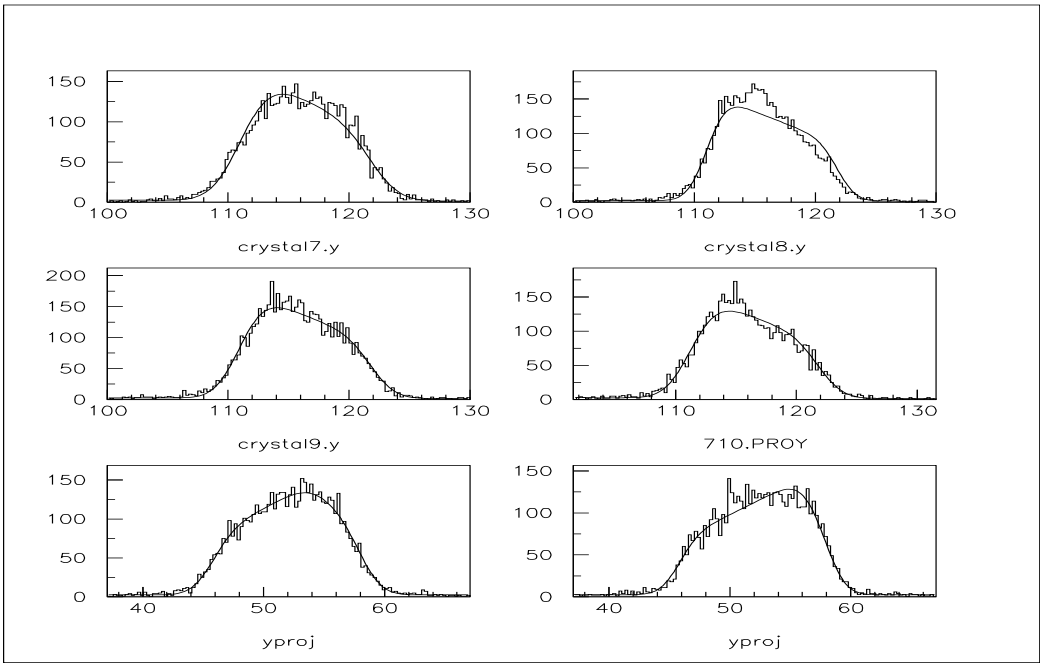
Also the choice of the parameters (and therefore trial functions) is not unique. For example for crystals D and B we could have fixed one of the

parameters by introducing the theoretical difference between two points and replacing one of the parameters by the sum(difference) of the second parameter and the theoretical difference. Careful analysis has shown that reducing the number of parameters in such a way does not improve our fits significantly (and sometimes makes it worse) but we loose half of the significant information.

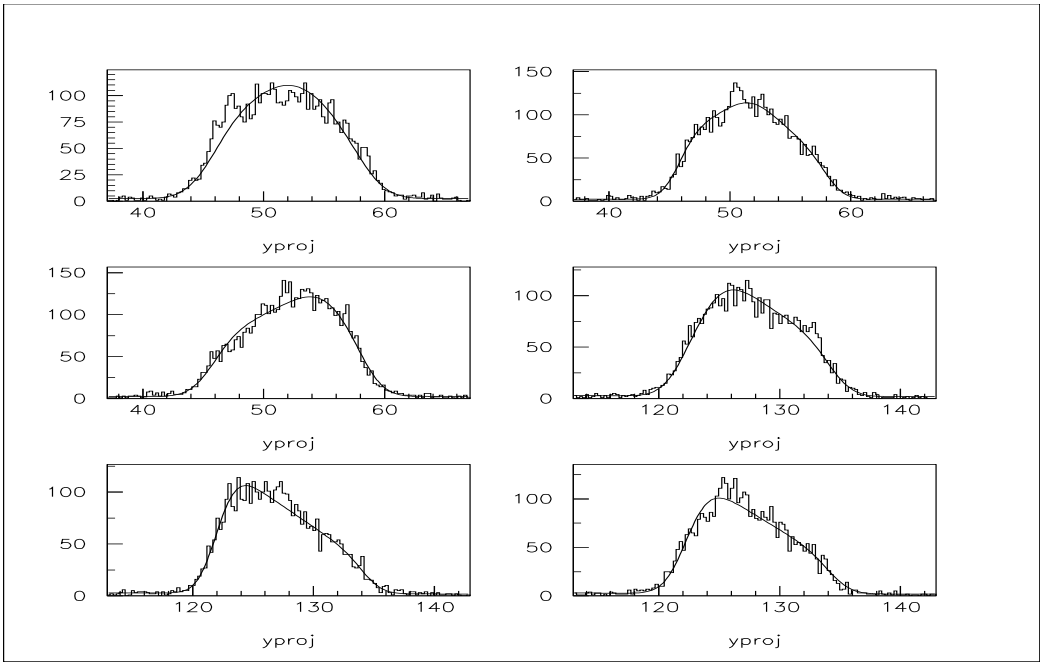
All the θ and typical φ projection fits can be seen on the picture below.



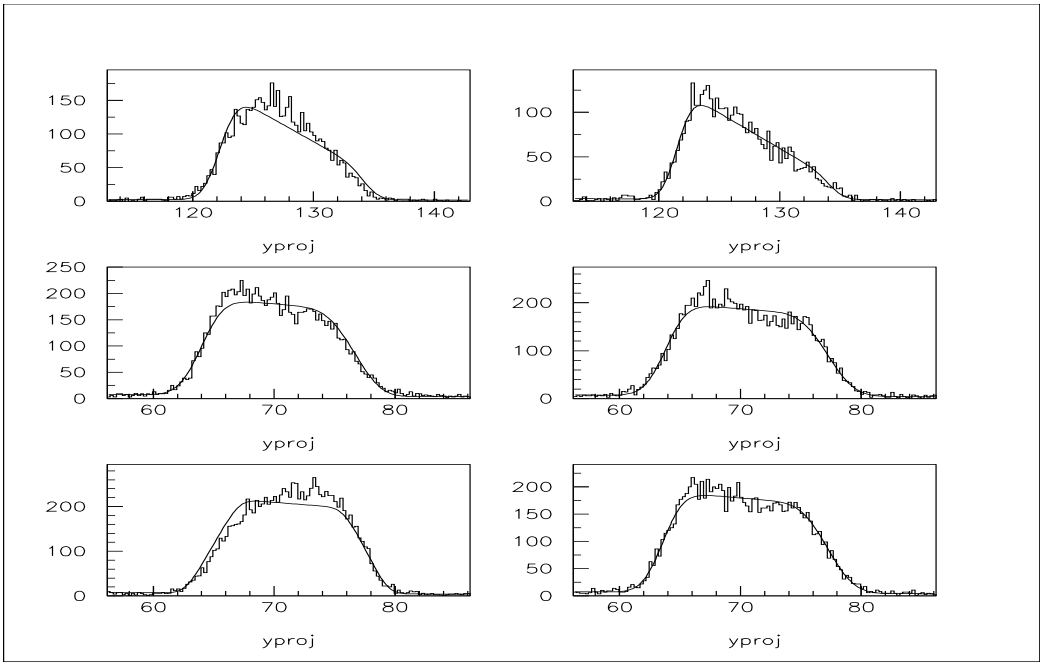
θ-projection fits for crystals 0 through 5



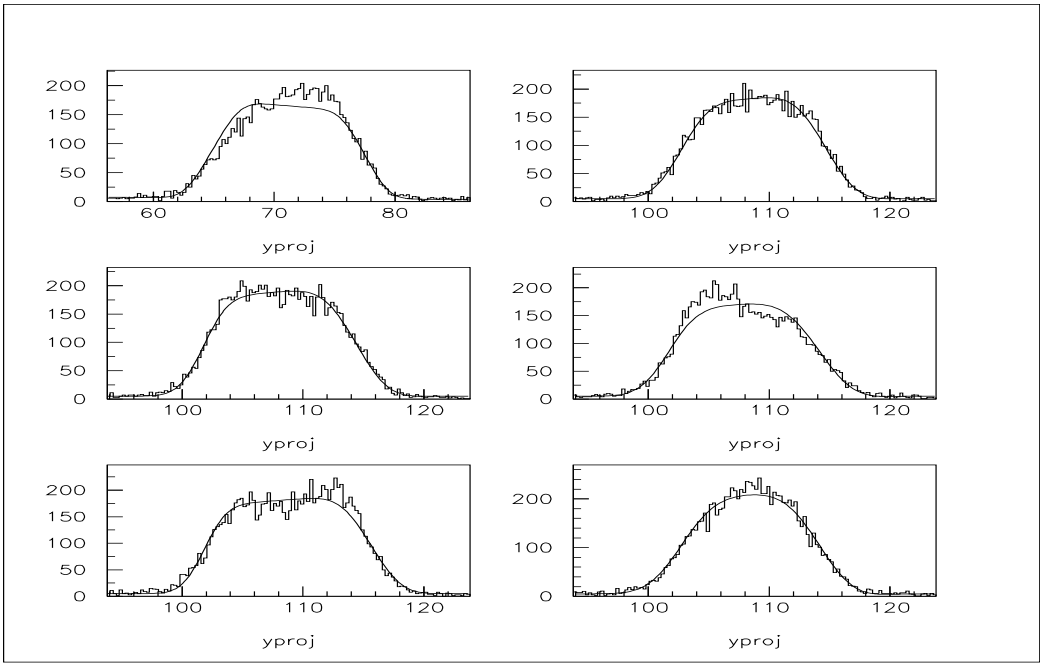
θ-projection fits for crystals 6 through 11



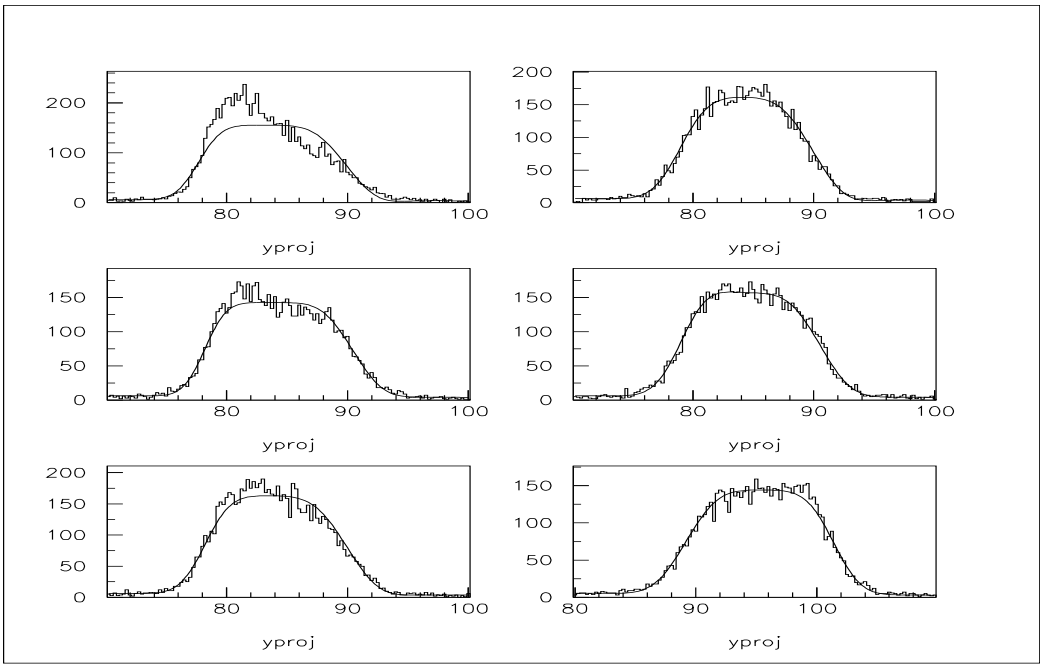
θ-projection fits for crystals 12 through 17



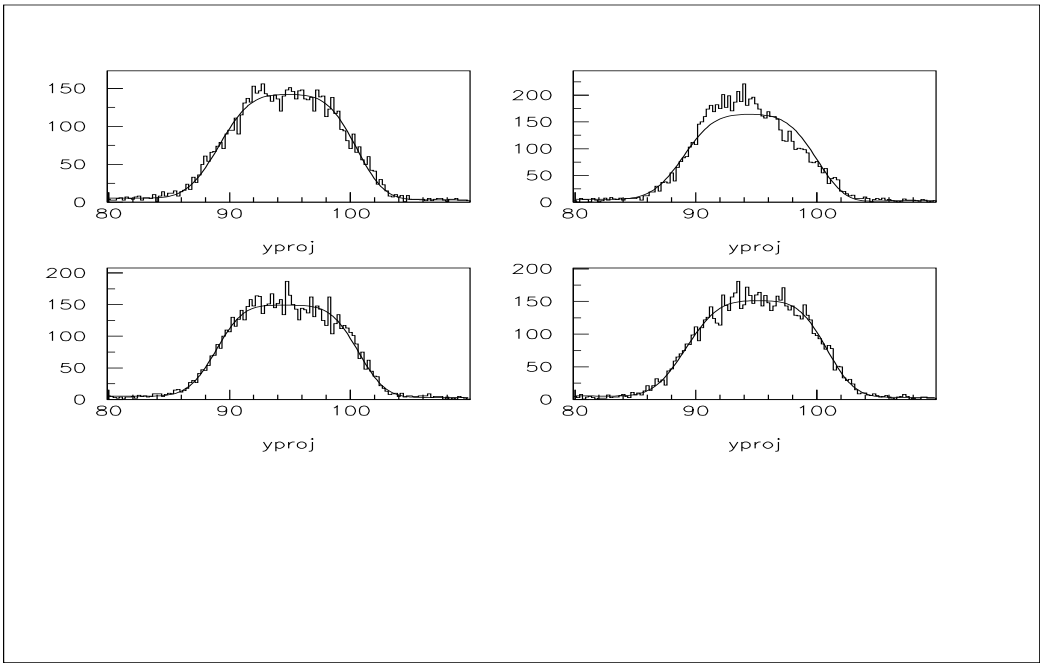
θ-projection fits for crystals 18,19 and 170 through 173



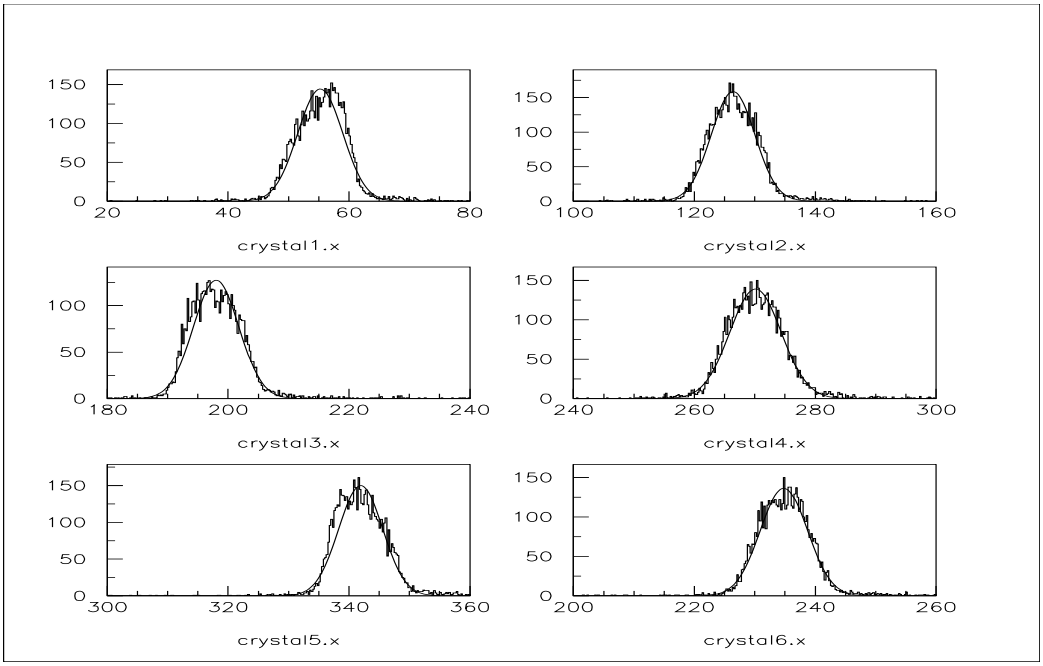
θ-projection fits for crystals 174 through 179



θ-projection fits for crystals 80 through 85

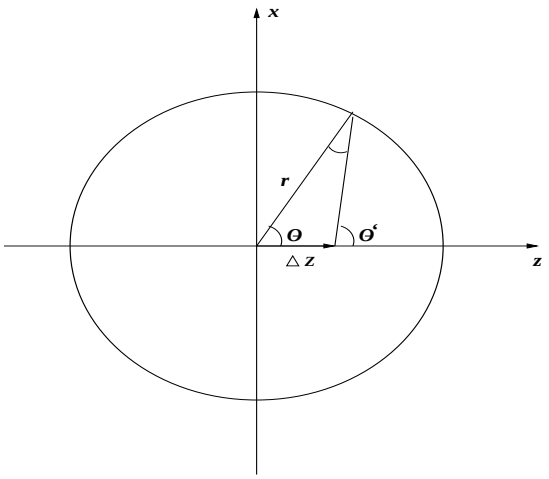


θ-projection fits for crystals 86 through 89



φ-projection fits for crystals 0 through 5

Compared to the theoretical values, all the φ angles look very close to expected values. It made us believe that there is no significant displacement of two systems in x or y directions. Assuming that only z displacement took place simple geometrical calculations yield the following result



Geometric setup for calculating displacement along z-axis

then

$$\Delta z > 0, \quad \Delta z = r\frac{sin(\theta^{'}-\theta)}{sin(\pi-\theta^{'})}$$
$$\Delta z < 0, \quad \Delta z = r\frac{sin(-\theta^{'}+\theta)}{sin(\theta^{'})}$$

where $\theta^{'}$ are angles deduced from the fits and θ are theoretical angles.

It is probably worth mentioning that since $sin(180-\theta) = sin(\theta)$ for any theta between 0 and π, formulae for positive and negative displacements look absolutely the same (clearly up to a minus sign).

5 Results

For all the analyzed crystals the chart of derived quantities included below.

$Crystal$	φ_{meas}	σ_{φ}	χ^2	θ_{meas}	σ_{θ}	χ^2	$\Delta z(cm)$	$\sigma_{\Delta z}(cm)$
1.	55.24	0.05	2.50	65.12	0.07	1.87	−0.28	0.04
2.	126.42	0.03	1.98	66.87	0.10	1.81	0.59	0.04
3.	198.04	0.04	2.18	65.15	0.08	2.46	−0.26	0.04
4.	270.14	0.04	1.73	64.70	0.07	1.66	−0.49	0.4
5.	341.95	0.03	2.96	65.36	0.10	1.08	−0.16	0.03
6.	234.81	0.05	1.60	112.17	0.16	1.31	−1.06	0.08
7.	306.14	0.04	1.69	111.33	0.08	1.33	−1.46	0.04
8.	19.31	0.05	2.45	111.26	0.08	2.00	−1.49	0.03
9.	90.59	0.06	2.37	111.17	0.06	1.23	−1.54	0.03
10.	162.34	0.04	1.38	111.68	0.08	1.30	−1.29	0.04
11.	54.55	0.07	1.67	54.47	0.27	0.89	1.09	0.15
12.	127.07	0.05	1.80	56.54	0.34	1.22	2.17	0.18
13.	198.19	0.06	2.17	52.55	0.30	1.57	0.02	0.17
14.	270.15	0.08	1.71	51.24	0.20	1.04	−0.74	0.12
15.	342.83	0.06	2.42	55.25	0.29	1.19	1.51	0.16
16.	233.61	0.07	2.50	124.39	0.25	1.04	−1.71	0.13
17.	306.45	0.08	1.67	122.17	0.08	1.08	−2.85	0.04
18.	18.55	0.09	2.20	122.82	0.16	1.33	−2.52	0.08
19.	90.32	0.07	1.98	122.68	0.12	2.56	−2.59	0.06
20.	162.58	0.08	1.36	121.29	0.05	1.38	−3.29	0.02
81.	54.52	0.05	2.04	77.68	0.05	4.76	−1.32	0.02
82.	127.79	0.03	3.30	80.55	0.84	1.37	0.01	0.38
83.	198.96	0.05	2.10	78.33	0.09	1.76	−1.01	0.04

84.	270.00	0.05	3.20	81.39	0.98	1.16	0.40	0.45
85.	341.84	0.03	3.75	78.35	0.11	1.39	−1.00	0.05
86.	234.23	0.06	4.23	99.85	0.07	0.80	0.17	0.03
87.	306.26	0.03	2.39	98.92	0.07	1.25	−0.26	0.03
88.	18.00	0.05	2.72	98.42	0.05	2.79	−0.49	0.02
89.	90.21	0.04	2.49	99.09	0.07	1.31	−0.18	0.03
90.	162.52	0.03	2.15	99.28	0.07	1.53	−0.09	0.03
81.	54.52	0.05	2.04	87.55	0.08	4.76	−0.48	0.04
82.	127.79	0.03	3.30	87.62	0.07	1.37	−0.45	0.03
83.	198.96	0.05	2.10	88.21	0.08	1.76	−0.18	0.04
84.	270.00	0.05	3.20	88.30	0.10	1.16	−0.14	0.04
85.	341.84	0.03	3.75	87.68	0.07	1.39	−0.42	0.03
86.	234.23	0.06	4.23	91.39	0.60	0.80	−0.00	0.27
87.	306.26	0.03	2.39	90.47	0.53	1.25	−0.42	0.24
88.	18.00	0.05	2.72	89.90	0.18	2.79	−0.68	0.08
89.	90.21	0.04	2.49	89.64	0.19	1.31	−0.80	0.09
90.	162.52	0.03	2.15	91.38	0.76	1.53	−0.01	0.34
171.	19.07	0.05	5.25	64.24	0.08	1.55	−1.47	0.04
172.	90.51	0.04	3.40	64.00	0.07	1.88	−1.59	0.03
173.	161.95	0.03	2.95	67.28	0.08	3.21	0.06	0.04
174.	234.74	0.03	3.99	63.69	0.10	1.30	−1.75	0.05
175.	306.61	0.03	2.20	67.26	0.07	2.20	0.05	0.03
176.	198.17	0.03	2.89	112.36	0.60	1.31	−0.24	0.30
177.	270.64	0.03	2.56	112.13	0.53	1.57	−0.36	0.26
178.	342.51	0.03	3.28	111.81	0.18	2.08	−0.51	0.08
179.	54.44	0.05	3.50	113.44	0.19	2.03	0.29	0.09
180.	125.98	0.03	3.62	111.77	0.76	1.56	−0.53	0.37
171.	19.07	0.05	5.25	74.69	0.06	1.55	−0.33	0.03
172.	90.51	0.04	3.40	75.26	0.08	1.88	−0.06	0.04
173.	161.95	0.03	2.95	75.68	0.06	3.21	0.14	0.03
174.	234.74	0.03	3.99	75.24	0.07	1.30	−0.06	0.03
175.	306.61	0.03	2.20	75.58	0.07	2.20	0.10	0.03
176.	198.17	0.03	2.89	105.08	0.54	1.31	0.22	0.25
177.	270.64	0.03	2.56	101.91	0.08	1.57	−1.26	0.04
178.	342.51	0.03	3.28	101.78	0.07	2.08	−1.32	0.03
179.	54.44	0.05	3.50	102.03	0.09	2.03	−1.20	0.04
180.	125.98	0.03	3.62	104.50	0.34	1.56	−0.06	0.16

In final calculations of weighted average for Δz I dropped crystals number 2,3,12 and 179 because despite the good statistical fits they didn't show expected signatures. It can be clearly seen from comparison of these crystals and their alike. The final result yielded

$$\Delta z = (-0.78 \pm 0.01)cm$$

thus assuring us that the chambers were "pushed" into the beam by above indicated amount.

6 Conclusions and possible improvements

Although this analysis provides us with important information with desired accuracy there is room for improvements. If we had better tracking procedure which uniquely associates each track with the crystal it hit, I could drop the requirement on keeping only one track and significantly improve the statistic for given set of data. Secondly, one could desire to have information about each individual crystal and its exact location with respect to the chambers. This would require more work and far more superior statistic since it could be seen that sometimes even the sign of the displacement does not agree within a crystal.